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Reg.No.:												

VIVEKANANDHA COLLEGE OF ENGINEERING FOR WOMEN
[AUTONOMOUS INSTITUTION AFFILIATED TO ANNA UNIVERSITY, CHENNAI]
Elayampalayam – 637 205, Tiruchengode, Namakkal Dt., Tamil Nadu.

Question Paper Code: 2010

B.E. / B.Tech. DEGREE END-SEMESTER EXAMINATIONS – DEC.2022 / JAN. 2023
Second Semester
U19MA202 – LINEAR ALGEBRA AND ORDINARY DIFFERENTIAL EQUATIONS
Common to All Branches
(Regulation 2019)

Time: Three Hours | Maximum: 100 Marks

Answer ALL the questions

Knowledge Levels (KL)	K1 – Remembering	K3 – Applying	K5 - Evaluating
	K2 – Understanding	K4 – Analyzing	K6 - Creating

PART – A

(10 x 2 = 20 Marks)

Q.No.	Questions	Marks	KL	CO
1.	If the eigenvalues of the matrix A of order $3X3$ are $2,3$ and 1, then find the eigenvalues of adjoint of A.	2	K1	CO1
2.	Find the nature of the quadratic form $2x^2 + 2xy + 3y^2$.	2	K3	CO1
3.	Find $grad\phi$ when ϕ is given by $\phi = log(x^2 + y^2 + z^2)$	2	K5	CO2
4.	If $\phi(x, y, z)$ is a scalar field, find$div(grad\phi)$.	2	K2	CO2
5.	Use Green's theorem to evaluate $\int_C (x^2 + xy)dx + (x^2 + y^2)dy$ where C is the square formed by the lines $x = \pm 1, y = \pm 1$.	2	K5	CO3
6.	Apply Green's theorem to evaluate the area of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$	2	K3	CO3
7.	Write the necessary and sufficient condition for the function to be analytic in polar coordinates.	2	K1	CO4
8.	Find the image of $2x + y = 3$ under the transformation $w = z + 2i$.	2	K2	CO4

9.	Find the Laplace Transform of $\frac{\delta(t-\pi)}{t}$.	2	K5	CO5
10.	Does the Laplace Transform of the function e^{t^2} exist? If not, why?	2	K4	CO5

PART – B

(5 x 16 = 80 Marks)

Q.No.			Questions	Marks	KL	CO
11.	a)	i.	Using Cayley-Hamilton theorem, find A^{-1} if $\begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$	8	K1	CO1
		ii.	Diagonalize the matrix $\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$	8	K2	
			(OR)			
	b)		Reduce the Quadratic form $3x_1^2 + 5x_2^2 + 3x_3^2 - 2x_1x_2 + 2x_1x_3 - 2x_2x_3$ to canonical form through an orthogonal transformation.	16	K3	CO1
12.	a)	i.	Find the angle between the surfaces $x^2 + y^2 + z^2 = 9$ and $z = x^2 + y^2 - 3$ at the point $(2,-1,2)$.	8	K1	CO2
		ii.	Prove that $div(gradr^n) = \nabla^2(r^n) = n(n+1)r^{n-2}$ where $\vec{r}=x\vec{i}+y\vec{j}+z\vec{k}$. Also, find $\nabla^2(\frac{1}{r})$.	8	K2	
			(OR)			
	b)	i.	Find the directional derivative of the function $f = x^2 - y^2 + 2z^2$ at the point $P(1,2,3)$ in the direction of the line PQ where Q is the point $(5,0,4)$.	6	K4	CO2
		ii.	A vector field is given by $\vec{F} = (x^2 + xy^2)\vec{i} + (y^2 + x^2y)\vec{j}$. Check whether the vector is solenoidal or irrotational and hence find its scalar potential.	10	K2	
13.	a)		State the Divergence theorem. Verify divergence theorem for $\vec{F} = (x^2 - yz)\vec{i} + (y^2 - zx)\vec{j} + (z^2 - xy)\vec{k}$ taken over the rectangular parallelopiped $0 \le x \le a, 0 \le y \le b, 0 \le z \le c$.	16	K3	CO3
			(OR)			

	b)	State Stoke's theorem. Verify Stoke's theorem for the vector $\vec{F} = z\vec{i} + x\vec{j} + y\vec{k}$ taken over the half of the sphere $x^2 + y^2 + z^2 = a^2$ lying above the $xy - plane$.	16	K3	CO3
14.	a)	i. Prove that an analytic function with constant real part is constant.	4	K1	CO4
		ii. Check whether the function $u = \frac{x}{x^2+y^2}$ is harmonic or not. If yes, find its conjugate harmonic and the corresponding analytic function $f(x,y) = u + iv$.	12	K3	
		(OR)			
	b)	i. Find the fixed points of the transformation $w = \frac{1+z}{1-z}$.	4	K1	CO4
		ii. Find the bilinear transformation which maps the points $z = 0, -1, i$ onto $w = i, 0, \infty$. Also find the image of the unit circle $\|z\| = 1$.	12	K3	
15.	a)	i. Find the inverse Laplace Transform of $log\left(\frac{s+1}{s-1}\right)$.	8	K2	CO5
		ii. Find the Laplace Transform of the triangular wave function of periodicity 2π, given by $f(t) = \begin{cases} t, & 0 < t < \pi \\ 2\pi - t, & \pi < t < 2\pi \end{cases}$.	8	K3	
		(OR)			
	b)	i. Using initial value theorem for the Laplace Transform, find $\lim_{t \to 0}(1 + e^{-2t})$.	4	K1	CO5
		ii. Solve the following differential equation using Laplace Transform $y''(t) + 2y'(t) + 5y(t) = e^{-t}sint$, where $y(0) = 0, y'(0) = 1$.	12	K3	